Lecture 1: The Cosmological Principle

Graduate Course in Astroparticles and Cosmology

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Plan of the lecture

1. What cosmology seeks to describe
2. The cosmological principle
3. Expansion of the Universe
4. Cosmological redshift
5. Synthesis

Suggested references and resources

Online lectures and notes

- Wayne Hu — *Cosmology Tutorials*
  `https://background.uchicago.edu/~whu/physics/physics.html`
- Hans Winther — *Modern Cosmology Notes*
  `https://cmb.wintherscoming.no/`
- Valery Rubakov - *Cosmology and dark matter* 1912.04727

Textbooks

- Steven Weinberg — *Cosmology*
- Scott Dodelson — *Modern Cosmology*
- Lars Bergström & Ariel Goobar - *Cosmology and Particle Astrophysics*

Cosmology as a physical science

Cosmology is the study of the Universe on the largest observable scales: its origin, large-scale structure, dynamical evolution, and possible fate. Unlike most branches of physics, it deals with a single system, observed at one stage of its history.

The theoretical problem is sharply posed:

Central question

What spacetime geometry and what forms of matter-energy describe the observable Universe, once one averages over sufficiently large scales?

The subject stands at the intersection of

- general relativity,
- statistical physics and thermodynamics,
- particle physics,
- astronomical observation.

Why a simple model is possible

It is remarkable that the large-scale Universe can be described by a small number of macroscopic variables, despite the enormous microscopic complexity of its contents.

The simplification rests on one assumption of decisive importance:

Cosmological Principle

On sufficiently large scales, the Universe is statistically homogeneous and isotropic.

This assumption is not exact on the scale of galaxies, clusters, or voids. It is an asymptotic statement about scales of order

$$\lambda \gtrsim 100\text{–}300 \ \mathrm{Mpc}. \tag{1}$$

Homogeneity and isotropy

Isotropy

The statistical properties of matter and radiation are independent of direction.

Homogeneity

The statistical properties of matter and radiation are independent of position.

- Isotropy about every point implies homogeneity.
- Homogeneity alone does *not* imply isotropy.
- Observed isotropy about us, together with the Copernican principle, motivates the large-scale homogeneous description.

Logical point: symmetry is not an aesthetic choice; it is the assumption that makes a predictive relativistic cosmology possible.

Observational support for the principle

- The cosmic microwave background is nearly isotropic, with residual temperature fluctuations at the level

$$\Theta \equiv \frac{\Delta T}{T} \sim 10^{-5} \tag{2}$$

  once the kinematic dipole is removed.
- Galaxy redshift surveys show rich inhomogeneous structure on small scales, yet approach statistical uniformity when averaged over sufficiently large volumes.
- Radio sources and distant quasars exhibit no compelling large-angle preferred direction.

Interpretation

The Universe is not smooth in detail; it is smooth in the statistical sense relevant for its large-scale dynamics.

CMB blackbody spectrum and near-isotropy

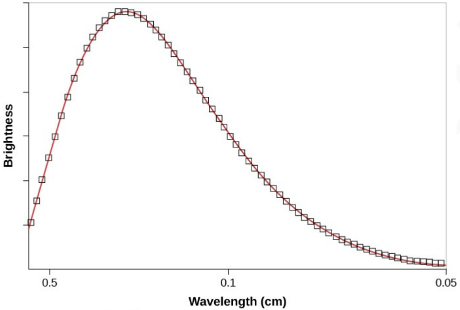

Figure: The solid line shows how the intensity of radiation should change with wavelength for a blackbody with a temperature $T_{\rm CMB} = 2.725\,{\rm K}$. The boxes show the CMB intensity as measured at various wavelengths by COBE's instruments. COBE confirmed smallness of anisotropies and thermal equilibrium. From https://courses.lumenlearning.com/suny-astronomy.

Hubble's law

The first great observational fact of modern cosmology is the recession of distant galaxies:

$$v = H_0 d. \tag{3}$$

Here

- v is the recession velocity,
- d is the physical distance,
- H_0 is the present Hubble constant.

Current determinations lie in the approximate range

$$H_0 \simeq 67\text{–}74 \ \mathrm{km\ s^{-1}\ Mpc^{-1}}. \tag{4}$$

Physical meaning

The linear law suggests not an explosion into pre-existing space, but a coherent expansion of space itself.

The geometry compatible with large-scale symmetry

Homogeneity and isotropy severely restrict the possible spacetime geometry. The unique line element consistent with these symmetries is the Friedmann-Lemaître-Robertson-Walker metric:

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\left[\frac{\mathrm{d}r^2}{1-Kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)\right]. \tag{5}$$

Its ingredients are:

- t: cosmic time,
- $a(t)$: scale factor,
- (r, θ, ϕ): comoving coordinates,
- $K = +1, 0, -1$: spatial curvature.

This is the kinematical backbone of all subsequent cosmology.

Spatial curvature

The sign of K determines the geometry of spatial slices:

$$K = \begin{cases} +1, & \text{closed Universe, positive curvature,} \\ 0, & \text{spatially flat Universe,} \\ -1, & \text{open Universe, negative curvature.} \end{cases} \tag{6}$$

With the normalization $K = \pm 1, 0$, the curvature radius of curved spatial slices is proportional to the scale factor:

$$R_{\rm curv}(t) = a(t) \qquad (K = \pm 1). \tag{7}$$

On scales much smaller than $R_{\rm curv}$, spatial geometry appears $\approx$ Euclidean.

Closed, flat, and open spatial geometries

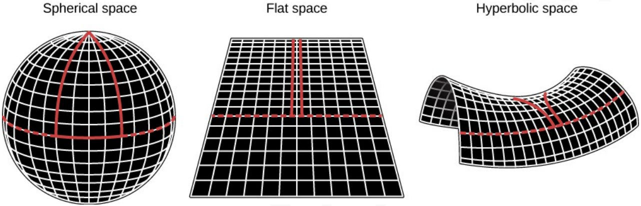

Figure: Possible spatial geometries of the Universe determined by the cosmic density relative to the critical density: closed ($K = +1$), flat ($K = 0$), and open ($K = -1$). In curved spaces initially parallel geodesics may converge or diverge depending on the sign of the curvature. From https://courses.lumenlearning.com/suny-astronomy.

Conformal time and comoving observers

It is often convenient to define the conformal time η by

$$\mathrm{d}\eta = \frac{\mathrm{d}t}{a(t)}. \tag{8}$$

Then the metric becomes

$$\mathrm{d}s^2 = a^2(\eta)\left[-\mathrm{d}\eta^2 + \gamma_{ij}\,\mathrm{d}x^i\mathrm{d}x^j\right], \tag{9}$$

where γ_{ij} is the metric of a maximally symmetric three-dimensional space.

Interpretive remark

Comoving observers remain at fixed spatial coordinates. Their physical separations change only because the scale factor evolves.

Proper distance and the meaning of expansion

For two galaxies at fixed comoving separation r, the proper distance is

$$d(t) = a(t)\, r. \tag{10}$$

Differentiating with respect to cosmic time gives

$$\dot{d}(t) = \dot{a}(t)\, r = \frac{\dot{a}}{a} d(t). \tag{11}$$

This already suggests the local form of Hubble's law.

Conceptual point

Galaxies need not be imagined as moving through a static Euclidean background. The dynamical object is the metric itself.

Hubble's law from the metric

Using the proper distance relation $d(t) = a(t)r$, one obtains

$$v \equiv \dot{d} = \dot{a}\, r = \frac{\dot{a}}{a} d. \tag{12}$$

Evaluated today,

$$\left.\frac{\dot{a}}{a}\right|_{t=t_0} \equiv H_0, \tag{13}$$

so that

$$v = H_0 d. \tag{14}$$

Important caution

This v is the rate of change of proper distance in an expanding spacetime. It is not, in general, a peculiar velocity in the special-relativistic sense.

Definition of cosmological redshift

As light propagates through the expanding Universe, its wavelength is stretched. The redshift is defined by

$$1 + z \equiv \frac{\lambda_{\rm obs}}{\lambda_{\rm em}}. \tag{15}$$

In an expanding Universe this becomes

$$1 + z = \frac{a(t_{\rm obs})}{a(t_{\rm em})}. \tag{16}$$

Interpretation

At cosmological distances, redshift is most naturally understood as a manifestation of the changing scale factor rather than as an ordinary Doppler shift in fixed space.

Null geodesics and redshift

For a radially propagating photon, the FLRW line element satisfies $\mathrm{d}s^2 = 0$, so

$$\frac{\mathrm{d}t}{a(t)} = \frac{\mathrm{d}r}{\sqrt{1 - Kr^2}}. \tag{17}$$

Now consider two successive wave crests emitted at $t_{\rm em}$ and received at $t_{\rm obs}$. Because they traverse the same comoving path,

$$\frac{\delta t_{\rm obs}}{a(t_{\rm obs})} = \frac{\delta t_{\rm em}}{a(t_{\rm em})}. \tag{18}$$

Using $\lambda \propto \delta t$, one immediately finds

$$1 + z = \frac{a(t_{\rm obs})}{a(t_{\rm em})}. \tag{19}$$

Low-redshift limit and the Hubble law

For nearby galaxies, where $z \ll 1$, one may write approximately

$$z \simeq \frac{v}{c}, \tag{20}$$

so that the redshift-distance relation reproduces the linear Hubble law.

At large distances, however, this interpretation becomes inadequate. The robust statement is instead

$$1 + z = \frac{1}{a(t_{\rm em})} \qquad \text{if } a(t_0) = 1. \tag{21}$$

Warning

Speaking about "recession velocity" is useful but secondary. The primary observable is redshift, with the scale factor as the dynamical variable.

What has been established

1. On sufficiently large scales, the Universe is well described as statistically homogeneous and isotropic.
2. These symmetries imply the FLRW metric.
3. The scale factor $a(t)$ measures the global expansion of the Universe.
4. Hubble's law follows directly from the metric description.
5. Cosmological redshift is the observational trace of the evolving scale factor.

From Geometry to Dynamics

- So far: geometry fixed by symmetry (FLRW metric)
- We now ask: how does the scale factor $a(t)$ evolve?
- The answer is provided by General Relativity

Einstein field equations

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu} \tag{22}$$

Geometry

- $G_{\mu\nu}$ encodes spacetime curvature
- Built from the metric $g_{\mu\nu}$ and its derivatives

Matter

- $T_{\mu\nu}$ encodes energy density and pressure
- Determined by the matter content of the Universe

Interpretation

Spacetime geometry is determined by the distribution of energy and momentum.

Perfect Fluid Approximation

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu} \tag{23}$$

- ρ: energy density
- p: pressure
- u^μ: four-velocity, $u^\mu u_\mu = -1 \quad$ (signature $-, +, +, +$)

In the comoving frame:

$$T^\mu_{\ \nu} = \mathrm{diag}(-\rho, p, p, p) \tag{24}$$

Energy Conservation

$$\nabla_\mu T^{\mu\nu} = 0 \tag{25}$$

Assuming an FLRW metric and a comoving perfect fluid:

$$\dot{\rho} + 3H(\rho + p) = 0 \tag{26}$$

- $H = \dot{a}/a$ is the Hubble parameter
- Encodes energy conservation in an expanding Universe

Thermodynamic Interpretation

$$\mathrm{d}(\rho a^3) = -p\,\mathrm{d}(a^3) \tag{27}$$

- Equivalent to $\dot{\rho} + 3H(\rho + p) = 0$
- Generalization of the first law: $\mathrm{d}E = -p\,\mathrm{d}V$
- Expansion performs work against pressure

Key point

This equation determines how the energy density evolves *once the relation between p and ρ is specified.*

Equation of State

From microphysics to cosmology

The Einstein equations require only the energy density ρ and pressure p. On large scales, the detailed microphysics of each component can be encoded in a relation between them.

A simple and widely applicable parametrization is

$$p = w\rho, \tag{28}$$

where w is the *equation-of-state parameter*.

- w characterizes how a component responds to expansion
- It determines how the energy density evolves with $a(t)$

Matter	$w = 0$
Radiation	$w = 1/3$
Vacuum energy	$w = -1$

Evolution of Energy Density

From the continuity equation:

$$\dot{\rho} + 3H(\rho + p) = 0 \tag{29}$$

we obtain:

$$\rho \propto a^{-3(1+w)} \tag{30}$$

- Matter (baryonic/dark): $\rho_m \propto a^{-3}$
- Radiation: $\rho_r \propto a^{-4}$
- Vacuum: $\rho_\Lambda = \mathrm{const}$

First Friedmann Equation

From Einstein equations to cosmology

Insert the FLRW metric and a perfect fluid energy-momentum tensor into

$$G_{\mu\nu} = 8\pi G \, T_{\mu\nu}. \tag{31}$$

Homogeneity and isotropy reduce the problem to time-dependent quantities only.

The 00 component yields

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}. \tag{32}$$

- $H \equiv \dot{a}/a$: expansion rate
- ρ: total energy density
- K: spatial curvature

Curvature as an effective energy density

Starting from the Friedmann equation

$$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}, \tag{33}$$

we can rewrite it as

$$H^2 = \frac{8\pi G}{3}\left(\rho + \rho_K\right), \tag{34}$$

where we define

$$\rho_K \equiv -\frac{3K}{8\pi G a^2}. \tag{35}$$

- Curvature behaves as an effective contribution to the energy density
- It scales as $\rho_K \propto a^{-2}$

Important remark

Curvature is not a physical fluid: it reflects the geometry of spatial slices, but it can be treated formally as a component in the Friedmann equation.

Second Friedmann Equation

How it is obtained

Insert the FLRW metric and the perfect-fluid tensor

$$T^{\mu}_{\ \nu} = \mathrm{diag}(-\rho, p, p, p) \tag{36}$$

into Einstein's equations. The spatial components give the acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p). \tag{37}$$

- Pressure contributes to gravitation
- Accelerated expansion requires $\rho + 3p < 0$

Second Friedmann Equation

Starting from

$$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}, \tag{38}$$

differentiate with respect to time:

$$2H\dot{H} = \frac{8\pi G}{3}\dot{\rho} + \frac{2K}{a^3}\dot{a}. \tag{39}$$

Using the continuity equation

$$\dot{\rho} = -3H(\rho + p), \tag{40}$$

and $\dot{a} = aH$, one finds

$$\dot{H} = -4\pi G(\rho + p) + \frac{K}{a^2}. \tag{41}$$

Second Friedmann Equation (cont.)

Since

$$\frac{\ddot{a}}{a} = \dot{H} + H^2, \tag{42}$$

substituting the first Friedmann equation gives

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p). \tag{43}$$

- Positive pressure enhances gravitational attraction
- Negative pressure drives accelerated expansion

Accelerated expansion corresponds to $w < -1/3$

Friedmann Equation in Standard Form

From densities to dimensionless parameters

Define the critical density

$$\rho_{c0} \equiv \frac{3H_0^2}{8\pi G}, \qquad \Omega_i \equiv \frac{\rho_{i0}}{\rho_{c0}}, \qquad \Omega_K \equiv -\frac{K}{a_0^2 H_0^2}. \tag{44}$$

Using the scaling laws

$$\rho_r \propto a^{-4}, \quad \rho_m \propto a^{-3}, \quad \rho_K \propto a^{-2}, \quad \rho_\Lambda = \text{const}, \tag{45}$$

Setting $a_0 = 1$, the Friedmann equation can be written as

$$H^2(a) = H_0^2 \left(\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_K a^{-2} + \Omega_\Lambda\right). \tag{46}$$

Evaluating at $a = 1$: $\Omega_r + \Omega_m + \Omega_\Lambda + \Omega_K = 1$.

- Each term represents a distinct component driving the expansion
- The evolution of $H(a)$ is determined by which term dominates

Redshift dependence of the Hubble rate

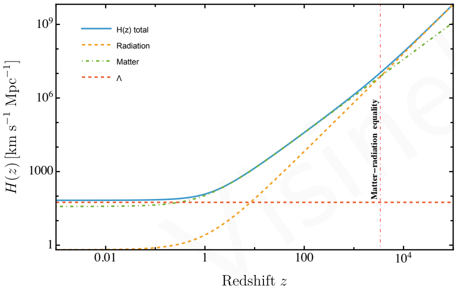

Figure: The Hubble rate $H(z)$ is decomposed into radiation, matter, and DE components. The change in slope across redshift reflects the transition between radiation domination at early times, matter domination at intermediate redshifts, and DE domination at late times. The vertical line at $z = z_{\rm eq}$ denotes matter–radiation equality, when $\rho_m = \rho_r$.

Summary

- Expansion governed by Einstein equations
- Matter content described as a perfect fluid
- Friedmann equations determine $a(t)$
- Different components dominate at different epochs