Lecture 2: Thermal History of the Universe

Graduate Course in Astroparticles and Cosmology

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Università di Salerno, Spring 2026

Plan of the lecture

1. The thermal universe
2. From the Hot Big Bang to ΛCDM
3. Key milestones in the early Universe
4. Recombination and the CMB

From expansion to thermal history

The Friedmann equations describe the global expansion of the Universe through the scale factor $a(t)$.

To understand the early Universe, however, one must also specify its *thermal state*.

Hot Big Bang picture

At early times the Universe was hot, dense, and filled with a nearly uniform plasma of elementary particles in approximate thermal equilibrium.

The central problem is to understand how

$$\text{expansion} \quad \leftrightarrow \quad \text{thermodynamics} \tag{1}$$

interact during cosmic evolution.

Hot Big Bang paradigm

At sufficiently high temperature, particle interactions are extremely rapid.

A species remains in thermal equilibrium as long as its interaction rate exceeds the Hubble rate:

$$\Gamma \gg H. \tag{2}$$

A rough estimate is

$$\Gamma \sim n\sigma v, \tag{3}$$

where n is the number density and σ the interaction cross section.

Physical picture

At early times collisions are frequent enough to maintain a thermal distribution; as the Universe expands and cools, interactions eventually become inefficient and species decouple.

Adiabatic cooling of the plasma

For an adiabatically expanding relativistic plasma, the entropy in a comoving volume is conserved:

$$sa^3 = \text{const.} \tag{4}$$

If the number of relativistic degrees of freedom is approximately constant, the temperature scales as

$$T \propto \frac{1}{a}. \tag{5}$$

Equivalently, in terms of the redshift,

$$T(z) = T_0(1+z), \qquad T_0 = 2.725\,\mathrm{K}. \tag{6}$$

Interpretation

Cosmic expansion stretches photon wavelengths, lowering the temperature of the radiation bath.

Radiation energy density

At sufficiently early times the energy density is dominated by relativistic species.

In thermal equilibrium,

$$\rho_r = \frac{\pi^2}{30} g_*(T) T^4. \tag{7}$$

The effective number of relativistic degrees of freedom is

$$g_*(T) = \sum_{\text{bosons}} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_{\text{fermions}} g_i \left(\frac{T_i}{T}\right)^4 . \tag{8}$$

Meaning of g_*

The effective number of relativistic d.o.f. counts all relativistic particle species contributing to the energy density, with fermions weighted by 7/8.

Entropy density

The entropy density of the plasma is

$$s = \frac{2\pi^2}{45} g_{*s}(T) T^3. \tag{9}$$

The entropy degrees of freedom are defined as

$$g_{*s}(T) = \sum_{\text{bosons}} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_{\text{fermions}} g_i \left(\frac{T_i}{T}\right)^3 . \tag{10}$$

In equilibrium and away from mass thresholds, one typically has

$$g_{*s}(T) = g_*(T). \tag{11}$$

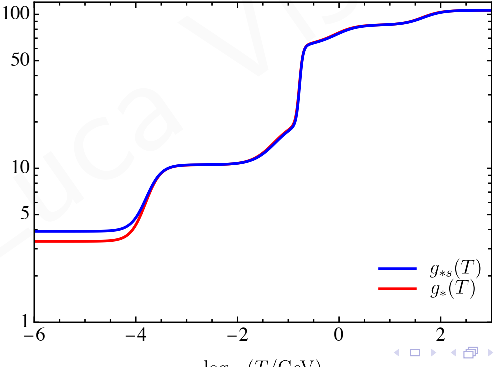
Remark

When particle species become non-relativistic, entropy is redistributed among the remaining relativistic species, and g_* and g_{*s} can differ slightly.

Evolution of $g_*(T)$ and $g_{*s}(T)$

As the Universe cools, particle species become non-relativistic and stop contributing efficiently to the energy and entropy densities.

- At $T \gtrsim 100\,\mathrm{GeV}$: all Standard Model species are relativistic
- $g_* = g_{*s} = 106.75$
- As T decreases: step-like reductions occur
- Near the QCD crossover: a rapid change takes place



Representative values of relativistic degrees of freedom

Epoch / temperature range	g_*	g_{*s}
$T \gtrsim 100$ GeV	106.75	106.75
$T \sim$ few GeV	61.75–75.75	61.75–75.75
$T \sim 150$–170 MeV	~ 17	~ 17
$T \sim 1$–10 MeV	10.75	10.75
after ν decoupling	6.863	7.409
$T \ll m_e$	3.363	3.909

The values decrease as heavy particles become non-relativistic and annihilate. After $e^{\pm}$ annihilation, photons and neutrinos remain, but with different temperatures.

Summary of the thermal picture

The thermal universe is governed by a simple interplay between expansion and microphysics:

- rapid interactions maintain thermal equilibrium,
- expansion cools the plasma,
- $g_*(T)$ and $g_{*s}(T)$ track the active particle content,
- temperature can be related directly to cosmic time.

The ΛCDM framework

The ΛCDM model provides a successful description of

- the expansion history of the Universe,
- the CMB anisotropies,
- the large-scale distribution of galaxies,
- the primordial abundances of light elements.

Its basic ingredients are:

- ordinary baryonic matter,
- cold dark matter,
- radiation,
- dark energy, modeled by a cosmological constant Λ.

Large-scale picture

The Universe is spatially very close to flat and evolves through a sequence of eras in which different components dominate the total energy density.

Radiation-dominated era

At sufficiently early times, the Universe is dominated by relativistic species:

$$\rho_r \gg \rho_m, \qquad p = \frac{\rho}{3}. \tag{12}$$

Because

$$\rho_r \propto a^{-4}, \qquad \rho_m \propto a^{-3}, \tag{13}$$

radiation dominates at small scale factor; tracing backward in time, radiation always wins.

The Friedmann equations then imply

$$a(t) \propto t^{1/2}, \qquad H = \frac{1}{2t}. \tag{14}$$

Importance

This era contains the main stages of the early thermal history, including decoupling processes and Big Bang nucleosynthesis.

Time–temperature relation

Combining the Friedmann equation with the radiation energy density gives

$$t \simeq 0.74\,\mathrm{s}\left(\frac{10.75}{g_*}\right)^{1/2}\left(\frac{1\,\mathrm{MeV}}{T}\right)^2. \tag{15}$$

Why this matters

This relation links the microscopic physics of the plasma to the age of the Universe.

It allows us to associate key events with definite times and temperatures, for example:

- neutrino decoupling,
- Big Bang nucleosynthesis,
- recombination.

Matter-dominated era

As the Universe expands, radiation redshifts away faster than matter.

After matter–radiation equality,

$$\rho_m \gg \rho_r. \tag{16}$$

The scale factor then evolves as

$$a(t) \propto t^{2/3}. \tag{17}$$

Physical consequence

This epoch is crucial for structure formation, because non-relativistic matter becomes the dominant source of gravity.

Dark-energy-dominated era

At late times the expansion is dominated by a component with negative pressure.

In the ΛCDM model, this is described by

$$\rho_\Lambda = \text{const.} \tag{18}$$

When dark energy dominates, the expansion approaches

$$a(t) \propto e^{H_\Lambda t}, \qquad H_\Lambda = \sqrt{\frac{\Lambda}{3}}. \tag{19}$$

Condition for acceleration

The second Friedmann equation shows that accelerated expansion requires

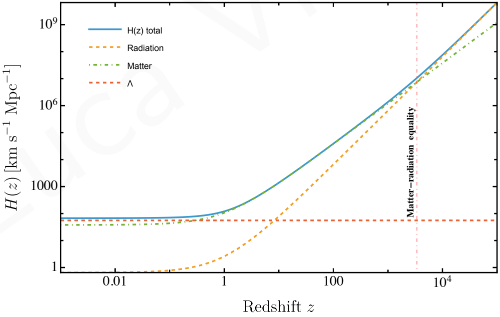
$$\rho + 3p < 0. \tag{20}$$

The Hubble rate evolves differently across the three main eras:

$$H^2(z) = H_0^2 \left[\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda\right]. \tag{21}$$

The change in slope reflects the transitions between:

- radiation domination,
- matter domination,
- dark-energy domination.



Age of the Universe

The present age follows from integrating the expansion history:

$$t_0 = \int_0^1 \frac{\mathrm{d}a}{aH(a)}. \tag{22}$$

For a flat ΛCDM cosmology,

$$t_0 = \frac{1}{H_0}\int_0^1 \frac{\mathrm{d}a}{a\left[\Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_\Lambda\right]^{1/2}}. \tag{23}$$

Using observed cosmological parameters, one finds

$$t_0 \simeq 13.8\,\mathrm{Gyr}. \tag{24}$$

Historical note

A purely matter-dominated universe with $\Omega_m = 1$ would give $t_0 = \frac{2}{3H_0}$, which is in tension with stellar age estimates.

Main thermal milestones

As the Universe cooled, several important transitions took place:

- Electroweak symmetry breaking: $T \sim 100\text{–}200\,\mathrm{GeV}$
- QCD confinement: $T \sim 200\,\mathrm{MeV}$
- Neutrino decoupling: $T \sim 1\,\mathrm{MeV}$
- Big Bang nucleosynthesis: $T \sim 0.1\,\mathrm{MeV}$
- Recombination: $T \sim 0.3\,\mathrm{eV}$

General rule

These events occur when interaction rates become comparable to the Hubble expansion rate:

$$\Gamma \sim H. \tag{25}$$

Neutrino decoupling and BBN

At temperatures around

$$T \sim 1\,\mathrm{MeV}, \tag{26}$$

weak interactions become too slow to keep neutrinos in equilibrium with the plasma.

Neutrinos then decouple and begin to free-stream through the Universe.

Shortly afterward, at

$$T \sim 0.1\,\mathrm{MeV}, \tag{27}$$

nuclear reactions synthesize the light elements during Big Bang nucleosynthesis.

Importance

BBN provides one of the earliest and most successful observational tests of standard cosmology.

After neutrinos decouple at $T \sim 1\,\mathrm{MeV}$, they no longer share entropy with the electromagnetic plasma.

When the temperature drops below $2m_e$, electron–positron pairs annihilate:

$$e^+ + e^- \to \gamma + \gamma. \tag{28}$$

Since neutrinos are already decoupled, this entropy release heats the photons but *not* the neutrinos.

Key consequence

The photon temperature becomes larger than the neutrino temperature:

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma. \tag{29}$$

As a result, the effective degrees of freedom for energy and entropy are no longer equal: $g_* \simeq 3.36, \qquad g_{*s} \simeq 3.91.$

Computing T_ν/T_γ from entropy conservation

For the species still coupled to photons, the entropy degrees of freedom are:

$$g_{*s}^{\rm before} = g_\gamma + \frac{7}{8} g_{e^\pm} = 2 + \frac{7}{8} \times 4 = \frac{11}{2}. \quad (30)$$

After annihilation, only photons remain in the electromagnetic bath:

$$g_{*s}^{\rm after} = g_\gamma = 2. \quad (31)$$

Entropy conservation in the photon sector implies

$$g_{*s}^{\rm before}\, T_{\gamma,\rm before}^3 a^3 = g_{*s}^{\rm after}\, T_{\gamma,\rm after}^3 a^3. \quad (32)$$

Since neutrinos are decoupled, $T_\nu \propto a^{-1}$, so comparing after annihilation gives

$$\frac{T_\nu^3}{T_\gamma^3} = \frac{g_{*s}^{\rm after}}{g_{*s}^{\rm before}} = \frac{2}{11/2} = \frac{4}{11}. \quad (33)$$

Photon coupling before recombination

Before recombination, matter and radiation formed a hot ionized plasma of photons, electrons, and baryons.

Photons scattered efficiently off free electrons via Thomson scattering:

$$\gamma + e^- \leftrightarrow \gamma + e^- . \tag{34}$$

The Thomson cross section is

$$\sigma_T = \frac{8\pi}{3}\frac{\alpha^2}{m_e^2}. \tag{35}$$

The interaction rate per photon is

$$\Gamma_\gamma = n_e \sigma_T c. \tag{36}$$

Tight coupling

At early times $\Gamma_\gamma \gg H$, so photons and baryons behave as a single interacting fluid.

Recombination

As the Universe cools, electrons and protons combine to form neutral hydrogen:

$$p + e^- \to \mathrm{H} + \gamma. \tag{37}$$

This process is called **recombination**.

Although the hydrogen binding energy is $13.6\,\mathrm{eV}$, recombination occurs much later, at

$$T \sim 0.3\,\mathrm{eV}, \tag{38}$$

because the photon-to-baryon ratio is extremely large:

$$\frac{n_\gamma}{n_b} \sim 10^9. \tag{39}$$

Consequence

A large number of energetic photons remains available to ionize hydrogen until the temperature becomes well below the binding energy.

Saha equation from chemical equilibrium (1)

Recombination is governed by the reaction

$$p + e^- \leftrightarrow H + \gamma. \tag{40}$$

In thermal and chemical equilibrium, the chemical potentials satisfy

$$\mu_e + \mu_p = \mu_H + \mu_\gamma. \tag{41}$$

Since photons are in equilibrium blackbody radiation,

$$\mu_\gamma = 0, \tag{42}$$

so the balance condition becomes

$$\mu_H = \mu_e + \mu_p. \tag{43}$$

For a non-relativistic species i, the number density is

$$n_i = g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} \exp\left[-\frac{m_i - \mu_i}{T}\right]. \tag{44}$$

Saha equation from chemical equilibrium (2)

Applying this to e, p, and H, and using $\mu_H = \mu_e + \mu_p$, one finds

$$\frac{n_e n_p}{n_H} = \frac{g_e g_p}{g_H} \left(\frac{m_e m_p}{m_H} \frac{T}{2\pi} \right)^{3/2} \exp\left[-\frac{m_e + m_p - m_H}{T} \right] . \qquad (45)$$

Binding energy

Since

$$m_e + m_p - m_H = E_I \simeq 13.6\,\mathrm{eV}, \qquad (46)$$

and $m_H \simeq m_p$, this becomes

$$\frac{n_e n_p}{n_H} \simeq \left(\frac{m_e T}{2\pi} \right)^{3/2} e^{-E_I/T}, \qquad (47)$$

which is the Saha equation.

Ionization fraction and Saha equation

Define the ionization fraction

$$x_e \equiv \frac{n_e}{n_b}, \tag{48}$$

where n_b is the baryon number density.

In equilibrium, x_e is approximately described by the Saha equation:

$$\frac{x_e^2}{1 - x_e} = \frac{1}{n_b}\left(\frac{m_e T}{2\pi}\right)^{3/2} e^{-13.6\,\mathrm{eV}/T}. \tag{49}$$

Interpretation

As the temperature drops, the exponential factor suppresses free electrons, and the ionized plasma rapidly becomes neutral.

Why the Saha equation matters

The Saha equation determines how the free-electron fraction x_e evolves as the Universe cools.

This has a direct impact on the photon interaction rate:

$$\Gamma_\gamma = n_e \sigma_T c \propto x_e. \qquad (50)$$

As x_e drops rapidly, the scattering rate decreases until

$$\Gamma_\gamma \sim H. \qquad (51)$$

Key consequence

The Saha equation explains the rapid decrease of the free-electron fraction. This decrease drives the Thomson scattering rate downward and makes photon decoupling possible, leading to the surface of last scattering.

- sets the redshift of recombination ($z \sim 1100$),
- determines the thickness of the last scattering surface,
- fixes initial conditions for CMB anisotropies.

Photon decoupling

As neutral hydrogen forms, the free-electron density decreases rapidly. The photon interaction rate $\Gamma_\gamma = n_e \sigma_T c$ also drops.

Photon decoupling occurs when

$$\Gamma_\gamma \simeq H. \tag{52}$$

Outcome

Photons cease to scatter efficiently and begin to propagate freely through the Universe.

This defines the **surface of last scattering**.

Surface of last scattering

The photons we observe today in the Cosmic Microwave Background were last scattered at recombination.

$$z_* \sim 1100, \qquad t_* \sim 3.8 \times 10^5 \, \mathrm{yr}. \tag{53}$$

Definition

The **surface of last scattering** is the set of spacetime points where photons last interacted with matter.

Important refinement

- decoupling is not instantaneous,
- this defines a surface with finite thickness,
- the width encodes the duration of recombination.

Interpretation

Observing the CMB corresponds to observing a spherical shell at

The Cosmic Microwave Background

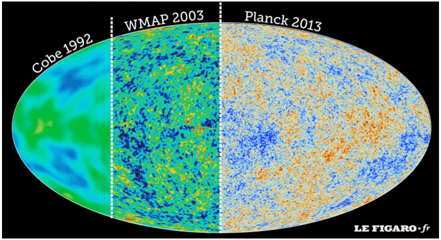
The photons released at decoupling constitute the Cosmic Microwave Background. Photon decoupling occurred at approximately

$$z_* \simeq 1100, \qquad t_* \simeq 3.8 \times 10^5\,\mathrm{yr}. \tag{54}$$

CMB today is observed as an almost perfect blackbody with temperature

$$T_0 = 2.725\,\mathrm{K}\,. \tag{55}$$

The CMB provides a direct snapshot of the Universe at recombination and contains information about the primordial perturbations.



From the thermal Universe to structure

We have followed the evolution of the Universe from a hot, dense plasma to the epoch of recombination, when photons decouple and form the Cosmic Microwave Background.

This picture describes an almost homogeneous and isotropic Universe, characterized by its thermal history and global expansion.

However, the real Universe contains galaxies, clusters, and large-scale structure.

Looking ahead

These structures originate from small primordial inhomogeneities, whose imprint is already visible in the CMB.
In the next lecture, we will study how primordial perturbations evolve and how they shape the observed anisotropies and large-scale structure.

Take-home messages

- The early Universe was a hot plasma in approximate thermal equilibrium.
- Radiation domination determines the relation between temperature, time, and expansion.
- As the Universe cools, key milestones occur: neutrino decoupling, BBN, and recombination.
- Recombination reduces the free-electron fraction and leads to photon decoupling.
- The CMB provides a snapshot of the Universe at $z \sim 1100$.
- In the next lecture, we will study how primordial perturbations are imprinted in the CMB anisotropies.